Is Geometry about Tangible Extension?

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In his *New Theory of Vision* (NTV)[1] Berkeley makes some comments about geometry that even he admits appear odd. He first writes what appears obvious—that "the constant use of the eyes, both in the practical and speculative parts of that science"—induces us to think that geometry is about visible extension. He further comments regarding those who note

> the extraordinary clearness and evidence of geometry, that in this science the reasonings are free from those inconveniences which attend the use of arbitrary signs, the very ideas themselves being copied out and exposed to view upon paper. (NTV 150)

Certainly in the 17th and 18th century geometry was considered a model for clear thinking.[2] But Berkeley's controversial point in NTV is that the common view is mistaken: both practical and speculative geometry are in truth about tangible extension. Visual diagrams—for example, constructions with straight edge and compass—are, as he says, "not even the likeness of figures which make the subject of the demonstration" (NTV 150). If there are geometric facts, they are learned by touch. Diagrams, Berkeley believes, are arbitrary signs of tactile information in the same way as written or spoken words are arbitrary signs of meanings. He writes:

> visible figures are of the same use in geometry that words are. And the one may be as well accounted the object of that science as the other; neither of them being otherwise concerned therein then as they represent or suggest to the mind the particular tangible figures connected with them. (NTV152)

By speculative geometry I take Berkeley here to mean Euclidean or classical geometry; and by practical geometry, geometry as applied, for example, in measurement. This paper takes issue with his claim that both kinds of geometry are ultimately about (in the sense of referring to) tangible extension; but my main interest is Berkeley's view of speculative geometry, particularly the question of how we should view the Euclidean postulates. More precisely, my thesis is that the visible lines, angles, and circles in geometrical diagrams are in fact the objects of classical geometry. They are thus not *merely* signs of geometrical properties that are in truth ascertained by touch.

Although Berkeley does argue (I think correctly) that the ability to do geometry (e.g., to describe lines and circles with straight edge and compass) requires tactile experience—thus a "disembodied" being possessed only of sight would lack such ability (NTV 153)—I propose that that claim is consistent with thinking classical geometry to be about the idealization of visible diagrams. Moreover, I will argue that Berkeley could have taken the basic terms of geometry such as "point," "line," etc. (as idealizations) to refer strictly to nothing at all.

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[1] George Berkeley, *New Theory of Vision* (NTV) (1732 edition, ed. Colin Murray Turbayne, Bobbs-Merrill, 1963).

[2] See Berkeley's comments in *The Analyst*, sec. 2, ed. A. A. Luce, in *Works*, vol. 4, eds. A. A. Luce and T. E. Jessop (London: Thomas Nelson and Sons, 1951).

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What then is Berkeley's argument for the view that tangible but not visible extension is the object of geometry? In NTV 151 he refers us back to sections 59−61 that demonstrate, he writes:

> that visible extensions in themselves are little regarded, and have no settled determinate greatness, and that men measure altogether, by the application of tangible extension to tangible extension. All which makes it evident that visible extension and figures are not the object of geometry.

But do those sections make that case? Below I consider the relative roles of touch and sight in elementary measurement. However, my main concern is with speculative geometry, for example, the postulates of Euclidean or classical geometry. Do sections 59, 60, and 61 show that the postulates, and consequently, the theorems, refer at bottom to what we learn through haptic experience?

Berkeley contends in NTV 59 that since the things we discover through touch rather than sight can hurt or help us, our main concern is with those tangible ideas signified by visual signs. Even if the reasoning were valid—and the fact that very bright lights can cause pain is perhaps a counter example to the premise—the argument by itself is certainly consistent with thinking visible extension is the object of classical geometry, in the sense that its postulates and theorems do refer, in his words "to the very ideas themselves being copied out and exposed to view upon paper" (NTV 150).

NTV 60 and 61 make the point that we judge the magnitude of an object not by its visible, but rather its tangible magnitude. We judge a man to be six feet tall, though visually he appears smaller and larger as we approach him. Contact by touch, on the other hand, provides us with the measure of an object invariant with respect to its visual appearance. The contact need not be by means of bodily contact, but indirect, for example, by laying a measuring rod along the edge of a table. As another example Berkeley notes that a "visible inch" is not a "determinate magnitude," since as we approach or move away from the ruler it will have more or less visible extension, or as he says, "more or less [visible] points discerned in it" (NTV 61). For practical purposes then, Berkeley would say that visual estimates of size by themselves are useless. Concluding NTV 61, Berkeley writes:

> Whenever we say an object is great or small, of this or that determinate measure, I say it must be meant of the tangible, and not the visible extension, which, though immediately perceived, is nevertheless little taken notice of.

I note two important points about NTV 61. First, though undoubtedly impractical, it doesn't seem impossible to have a non-contact metric; we pick a particular distance from an object and judge its height with a ruler in terms of how it visually appears from that distance. By convention we might take that to be its standard height. More convenient obviously is to take the spatial distance between object and measuring rod to be zero—for example, when we make the measuring rod coincident with the side of the object. Second, and important for some of my later discussion about classical geometry, Berkeley accepts in NTV that extension, whether visible or tangible, is composed of sensible minima. In NTV 61 his concern is minima visibilia. However, in NTV 54 he writes:

> Each of these magnitudes [visual and tangible] are greater or lesser, according as they contain in them more or fewer points, they being made up of points or minimums . . . There is a *minimum tangible*, and a *minimum visible*, beyond which sense cannot perceive.

There are certainly problems here. For example, you can't count, in an ordinary sense, the number of minima in a bit of extension. That requires recognizing boundaries between minima and therefore perceiving something less than a minimum. Moreover, thinking of sensible extension as non-continuous (i.e., composed of minima) raises the question whether the fundamental terms of classical geometry (e.g., point, line, plane, circle, etc.) refer at all to sensible extension. Again I address that below.

Of more significance here is the fact that nothing in Berkeley's remarks (which are about measurement) show that speculative geometry—by which again I mean classical Euclidean geometry (the kind we in the U.S. usually learn in tenth grade)—is, as Thomas Reid (echoing Berkeley) claimed, about the properties of tangible extension. Reid writes the following:

> Those figures and that extension which are the immediate objects of sight, are not the figures and the extension about which common geometry is employed; that the geometrician, while he looks at his diagram, and demonstrates a proposition, hath a figure presented to his eye, which is only a sign and representative of a tangible figure ... and that these two figures have different properties, so that what he demonstrates of the one, is not true of the other.[3]

According to Eduard Slowick, from whom I took the above quote, Reid (again following Berkeley) characterizes the geometry of physical objects—as opposed to a "geometry of visibles"—as Euclidean and revealed to touch; it gives the real as opposed to the apparent magnitude of a body.

However, it seems to me there are strong arguments that vision is essentially, not just peripherally involved in both practical and speculative geometry. Take elementary measurement: for example, determining the length of an object by laying a ruler successively along its edge. The judgment of congruence between the ruler and part of the edge seems clearly made by sight. Berkeley is correct that measurement gives a number invariant with respect to changing visual estimates of size as I approach an object. And that's certainly useful. But again it's by sight that we ascertain the congruence between a portion of the ruler and a doorway's edge *when* they are spatially contiguous.

Perhaps Berkeley would want to say that the judgment of congruence of ruler and edge is ultimately made by touch, the visible appearance of congruence being an arbitrary sign for that determination. Or, put another way, the apparently visual determination of congruence is a sign of what would be the case if we ascertained by touch the match between a ruler and an object's edge. But this choice has serious problems. As in other areas, touch is not always decisive for what we

[3] See Edward Slowik, "Conventionalism in Reid's Geometry of Visibles," *Studies in the History and Philosophy of Science*, Part A, 34 #3 (Sept. 2003), 470.

take to be the case. And in the above example of measuring the edge of an object, it's not even clear what it means to determine equal measure or congruence by touch. How is that done? In elementary measurement we might indeed speak of a contact perspective: that is, the ruler is laid alongside of an object's edge. But the judgment that a section of the edge is three ruler inches is made by sight.[4]

Furthermore, there are elementary examples where sight makes finer tuned judgments than touch—for example, distinguishing between objects that have elliptical boundaries. Some pairs of object judged correctly by sight to be respectively spherical and oval, will be judged by touch to have identical curvature. If Berkeley were right, the difference noted by sight should signify a tactually recognized difference.

One objection to the above discussion is that Berkeley's "heterogeneity" thesis—that the immediate (proper) objects of touch are distinct from the immediate objects of sight—rules out assuming that the object whose congruence with the ruler I determine by sight is numerically identical to the object whose congruence with the ruler is (somehow) determined by touch. But even to make sense of the Molyneux example as support for the heterogeneity thesis (NTV 132), Berkeley assumes that the *same two* objects that a congenitally blind subject by touch distinguishes as globe and cube can't by him, when he gains sight, be immediately (visually) distinguished as globe and cube. Without the assumption that objects can be re-identified over time by different senses, the example doesn't get off the ground. We assume that it's the s*ame* cube that the Molyneux man previously identified by touch that he later, after association between visual and tactile data, now can identify by sight. By allowing this assumption of object re-identification, we can then rightly note that we sometimes make finer discriminations between objects by sight than we make by touch. Therefore it's false that discriminations between objects apparently made by sight are simply signs of discriminations between *those same* objects that would be made by touch.[5]

Now it is true that even the congenitally blind can and do learn (and even teach) theoretical and applied geometry. Perhaps the most famous example is Nicolas Saunderson (1682-1739), third appointee to the Lucasian chair of Mathematics at Cambridge in 1711. Blinded by smallpox at the age of one, he taught Euclidean geometry as well as optics and algebra. And today there are a variety of creative methods for teaching elementary (Euclidean) geometry to blind high school students.[6]

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[4] See Roy Sorenson, *Seeing Dark Things: The Philosophy of Shadows* (Oxford: Oxford University Press, 2008), 123-24. NTV assumes, at least for the sake of presenting the argument against geometrical optics as a sufficient account of seeing distance, that we perceive by sight what we touch, although the proper objects of sight are wholly different from the proper objects of touch. However, without the assumption of a correlation between sight and touch, I think there is no coherent notion of measurement. I discuss this below in relation to the heterogeneity thesis.

[5] With Berkeley I wouldn't give too much importance to the word *same*. He often thinks it's a matter of convention whether we judge things to be numerically identical. But that convention or assumption I think is needed to make sense of the Molyneux case.

[6] See, for example, Patrick Roth and Lori Petrucci, "From Dots to Shapes," an auditory haptic game platform for teaching geometry to blind pupils**,** Thierry Pun Computer Science Department CUI, University of Geneva, Switzerland. I thank Gaylen Kapperman (Coordinator of Programs in Vision, Department of Teacher

A related interesting question is whether a blind person without the assistance of a sighted person would discover—as opposed to being taught—the postulates of Euclidean geometry. We can think of this as a variant thought experiment to Berkeley' conjecture in NTV 153 noted above about whether a being endowed only with sight could do geometry. Berkeley's answer to this question was no. Neither constructions with straight edge and compass nor superposition would be possible or even comprehensible for such a being. With respect to whether the blind could discover the Euclidean postulates, Diderot remarks about Saunderson:

> Now, it is obvious that however acute the blind man may be, the phenomena of light and colour are completely unknown to him. He will understand the axioms, because he refers them to palpable objects, but he will not understand why geometry should prefer them to other axioms, for to do so he would have to compare the axioms with the phenomena which for him is an impossibility.[7]

One way to read the passage is that Saunderson, not being able to see constructions made, for example, with straight edge and compass, will not take the axioms (postulates) of classical geometry to be self-evident. As I will later argue, those postulates won't have the intuitive power that Berkeley himself notes in *Analyst* 2. Assuming this reading, I think Diderot is correct. Take just the first postulate in one of its historically interesting versions, that no two straight lines enclose a space. Its intuitive self-evidence certainly appears to be given by sight. As Berkeley notes:

> when the postulata cannot be refused, nor the axioms denied; when the distinct contemplation and comparison of figures, their properties are derived, by a perpetual well-connected chain of consequences, the objects being still kept in view, and the attention ever fixed upon them; there is acquired a habit of reasoning, close and exact and methodical: which habit strengthens and sharpens the mind, and being transferred to other subjects is of general use in the inquiry after truth. (*Analyst* 2)

The passage raises two related questions: (1) What did Berkeley think the "postulata" that "can't be refused" were ultimately about? And (2) what "objects" does Berkeley think are "held in view" as a demonstration goes through? For Berkeley, at least in *The New Theory*, the common sense (but he believes incorrect) answer to both questions is the visible lines in the diagram. And I think he would agree that the intuitive certainty that in a plane no two-sided polygon exists appears *at first glance* to be grasped by sight. [8] It's this visual apprehension that Diderot suggests is forbidden to Saunderson. Yet, as I read NTV 150 and 151 Berkeley must take the intuitive

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Education, Northern Illinois University) for helpful discussions on teaching geometry to blind students. It is his view that blind students, though taught the Euclidean postulates, don't take them as self-evident.

[7] Denis Diderot, "Letter on the Blind, for the Benefit of Those Who See" (1749), trans. M. J. Morgan, in M. J. Morgan, *Molyneux's Question: Vision, Touch and the Philosophy of Perception* (Cambridge: Cambridge University Press, 1977), 31–58.

[8] One referee objected that a two-sided polygon is not conceivable much less visualizable. But it is certainly conceivable, depending of course on what that term means. Euclid's first postulate (viz., a straight line can be drawn between any two points) is not an analytic truth.

certainty (not necessarily the truth) of the postulate to be in fact revealed to touch,[9] since his other choice would be that classical geometry refers to properties of the diagrams, a position he rejects.

Though not decisive, I note that my brief research into how Euclid is taught to blind students supports what I take to be Diderot's point. Two considerations seem particularly relevant. First, teaching plane geometry to congenitally blind students is evidently extremely difficult. Second, although the postulates are taught obviously by means of haptic, and even auditory experience, that experience doesn't reveal them as self-evident.[10] Of course, self-evident doesn't mean true; and as applied to the physical world, where light rays, or longitudes on a globe stand in for straight lines, they are arguably false. The point I am making is they appear to sight, as Berkeley himself notes, as self-evident—that is, they have the *property* of being self-evident. If what we see is simply a guide to properties of an underlying tactile reality, then those properties should appear self-evident to touch.

In any case, Berkeley's view that extension, whether visible or tangible, is composed of minima rules out incommensurable line segments, and therefore (as he often remarks in the *Notebooks*), classical geometry is false for sensible extension. No drawn circle could be Euclidean since its circumference and diameter would be commensurable (a ratio of two whole numbers). That is how Berkeley can observe rather boastfully that he as opposed to others can square the circle (NB 249-50, 458, 511).

One possibility for preserving Euclidean geometry, based on remarks Berkeley makes in *De Motu* (DM), is that he could have taken all of classical geometry to be a useful fiction (even though he did not). In DM 39 he writes:

> And just as geometers for the sake of their art make use of many devices which they themselves cannot describe nor find in the nature of things, even so the mechanician makes use of certain abstract and general terms, imagining in bodies force, action, attraction, solicitation, etc. which are of first utility for theories and formulations, as also for computations about motion, even if in the truth of things, and in bodies actually existing, they would be looked for in vain, just like geometers' *fictions made by mathematical abstraction*. (DM 39, my emphasis)

Berkeley, I believe, could have taken all of Euclidean geometry to be fictional, just as he undoubtedly assumed the perfectly spherical balls and frictionless planes in Galileo's experiments on falling bodies[11] or mass points in Newton's *Principia*. In the same way, the terms "point," "line," and "plane" in geometry would be referentially empty, as is the phrase "mass point" in dynamics.

Of course, one might agree that Berkeley should have considered Euclidean geometry a useful

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[9] Douglas Jesseph quotes the passage to illustrate that, as opposed to what Berkeley writes in the *Notebooks*, in *The Analyst* he accepts or at least is more comfortable with classical geometry. See Douglas Jesseph, *Berkeley's Philosophy of Mathematics* (Chicago: University of Chicago Press, 1993), 84-85.

[10] I gathered this to be the case from conversations with Gaylen Kapperman.

[11] Galileo Galilei, *Dialogue on Two New Sciences* (1638), trans. Henry Crew and Alfonso de Salvio (Amherst: Prometheus Books, 1991), 169-72.

fiction and still think that its postulates are idealizations of what we experience through touch rather than sight. Take as an example, the famous fifth postulate in its modern version (John Playfair, 1745) that in a plane containing both a line and external point, there is exactly one line through the point parallel to the first line. That there is at least one line can be "demonstrated" using straight edge and compass. The use of these tools undoubtedly requires a sense of touch, but, in addition, the intuitive power (again not necessarily the truth) of the second part of the postulate—that there is only one such line—should, if Berkeley is correct, be gained from tactile experience. Remember that Berkeley takes the properties we claim to see in the diagram to be arbitrary signs for what's in truth revealed to touch. Yet direct touching seems unlikely to be fine-grained enough to give that result. Indirect touching, for example, tracing the boundaries of objects with a pencil or compass point, might work, but at present I know of no experiments with congenitally unsighted persons that test this. In any case, Berkeley's theory of vision appears to imply (using a thought experiment more realistic than his speculation about a purely sighted being) that a community of rational never-sighted persons unaided by those with vision would create Euclidean geometry as an idealization of their tactile and kinesthetic experience.[12] Again, this is simply because if (1) some postulates of classical geometry appear by sight to *have the property* of being self-evident, (2) geometry is not about the diagrams, and (3) properties we think possessed by the diagrams are in truth revealed to touch, then the blind community should find the postulates to be self-evident. However, I don't think such a community would likely find the classical postulates to be self-evident (though I don't know).[13]

I've argued elsewhere that even assuming Berkeley correctly identifies the origins of our apparently visual experience of space—that it results from an arbitrary but universal association of proper (immediate) visual sense data with immediate kinesthetic and tactile sense data—there could be new visual experiences, for example, of outness or spatial extension.[14] These experiences can be properly ascribed to sight rather than thought of as simply reading tactile *significata* through visual signs, analogous (Berkeley thought) to reading through script to underlying meaning. True, I might imagine tracing my finger around the boundary of a drawn triangle, and to that extent my vision is informed by tactile experience. And it may well be true that like Berkeley's purely sighted being, I couldn't even "see" that triangle without experiences gained through touch. As Berkeley points out, I certainly couldn't describe a line or a circle with a straight edge or compass. But it can be true as well that classical geometry is about properties (idealized) of the diagrams in Euclid's treatise.[15]

I'll close by briefly considering Margaret Atherton's recent discussion of some of the passages in

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[12] In his short story "The Country of the Blind," H. G. Wells suggests what such community would be like. See his *Country of the Blind and Other Selected Stories* (New York: Penguin Classics, 2007), 342-48.

[13] I have walked around the house with my eyes closed, running my fingers along edges, and it doesn't seem to me obvious that, for example, there's only one of what I would call a straight line between two of what I would call points. Admittedly, this is not a decisive experiment. Being sighted doesn't help.

[14] See R. Brook, "Berkeley's Theory of Vision: Transparency and Signification," *British Journal for the History of Philosophy* 11 (2003), 691-99.

[15] D. M. Armstrong appears to accept the inference that if a sense of touch is required to understand classical geometry then the latter is about tangible extension. See his *Berkeley's Theory of Vision* (Melbourne: Melbourne University Press, 1960), 58-59. That assumes, as I would not, that correlations between immediate (proper) visual and tactile experience can't change the phenomenal character of visual experience.

NTV dealt with here.[16] In a kind of summary of her view Atherton comments, "If, as Berkeley has argued, the proper subject matter of geometry does not include what we see, then the geometric theory of vision is trying to solve a false or non-existent problem" (206). Although the argument may be valid, we can accept the conclusion and reject the premise. That premise is that the proper subject of geometry is not what we see. My view is that "the very ideas themselves being copied out and exposed to view upon paper" (NTV 150)—idealized by being subject for Berkeley to the Euclidean formalism—are in fact the proper subject matter of classical geometry; while it remains true that geometrical optics fails to account sufficiently for how we see distance. And that I think is Atherton's major point.

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[16] Margaret Atherton, *Berkeley's Revolution in Vision* (Ithaca: Cornell University Press, 1990), 201-207.